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Applicability of the Bethe-Bloch Equation to Fast Heavy Charged Particles in Liquid Hydrogen

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液体水素中の高速重荷電粒子に対するベーテ・ブロッホ方程式の適用性

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概　要

液体水素中に入射する高速重荷電粒子に対して，小角散乱およびベーテ近似の有効なエネルギー領域を求めることにより，それらの近似の上に成立するベーテ・ブロッホ方程式の適用性を論じる。

物質中に於ける高速重荷電粒子のエネルギー損失（阻止能）は，その粒子と原子の束縛電子との非弾性散乱によってひき起こされ，ベーテ・ブロッホ方程式 (1) で表わされる。ここで，z, v は入射粒子の荷電数と速度，N, Z は物質の単位体積中の原子数と原子あたりの電子数（原子番号），m, e は電子の質量，電荷である。I は原子の平均イオン化ポテンシャルで，その原子の励起エネルギー E_n と，それに対応する比振動子強度 f_n とによって，(2) で表わされる。

運動量 p, 質量 M, 運動エネルギー E の入射粒子が原子との衝突によってうる反跳運動量 q は，完全弾性衝突を仮定した相対論的計算によると (3) で与えられる。m/M をパラメーターにとり，この関係式 $q/p-E$ のグラフを第1図に示す。小角散乱近似の有効基準として，反跳運動量範囲を $q/p<0.1$ に選び，同図中に点線で示す。この仮定によると，第1図より小角散乱近似の有効なエネルギー範囲は，陽子に対しては，100GeV 以下，μ 中間子に対しては 1 GeV 以下となる。

束縛電子と入射粒子との衝突断面積は，原子の励起エネルギー E_n と電子の反跳エネルギー Q との関数 $d\sigma(E_n, Q)$ で表わされる。この微分断面積への主要な寄与をする (E_n, Q)

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領域は，電子の平均運動エネルギーを $\langle k_0 \rangle$ とすると，$E_n < \langle k_0 \rangle$ の場合は $Q < \langle k_0 \rangle$，$E_n > \langle k_0 \rangle$ の場合は (4)（ベーテの尾根）で与えられる。水素分子に対し，$\langle k_0 \rangle$ を 30 eV に選び，運動量保存からえられる反跳エネルギーの下限 (5) を第2図に示す。阻止能は，$E_n d\sigma$ (E_n, Q) の (E_n, Q)－平面に於ける，力学的に許容される範囲 $Q_{\min} < Q < Q_{\max}$ にわたる積分 (6) で与えられる（ベーテ近似）。この近似は，電子の反跳エネルギーが (5) とベーテの尾根 (4) の下限とで囲まれる三カ月形の領域に含まれ，かつ $Q_1 < \langle k_0 \rangle$，$Q_2 \gg \langle k_0 \rangle$ であるような場合に成立する。第2図より，この条件を満す入射粒子のエネルギーは，陽子に対し，1 MeV 以上，μ 中間子に対し，100 keV 以上となる。この近似は入射粒子の運動エネルギーが高い程，より小さな Q_1，より大きな Q_2 が許されるので，一層良い近似となる。

したがって，液体水素中に入射した高速重荷電粒子に対して，ベーテ・ブロッホ方程式が適用できる入射エネルギー範囲は，陽子に対して，1 MeV $<$ E $<$ 100 GeV，μ 中間子に対して，100 keV $<$ E $<$ 1 GeV である。

Applicability of the Bethe-Bloch Equation to Fast Heavy Charged Particles in Liquid Hydrogen

By using the small angle scattering and Bethe approximations, the applicability of the Bethe-Bloch equation to incident fast heavy particles into liquid hydrogen are discussed. Adequate energy ranges for these particles by these approximations are given:
For protons and μ mesons they are 1 MeV $< E <$ 100 GeV and 100 keV $< E <$ 1 GeV, respectively.

An energy loss of fast charged particles in matter is induced by the inelastic collision with bound electrons of atoms. This energy loss is expected by so called Bethe-Bloch's equation, as follows;

$$-\frac{dE}{ds}=\frac{2z^2e^4}{mv^2}NZ\left\{ln\frac{(2mv^2)^2}{I^2}+2ln\frac{1}{1-\beta^2}-2\beta^2\right\}, \tag{1}$$

where mean ionization potential I is expessed by the equation

$$ln\ I=\sum_n f_n ln E_n, \tag{2}$$

where E_n is excitation energy and f_n is specific oscillator strength corresponding to E_n. And other notations are conventional.

In this paper we discuss first the efficiency of various approximations used in deriving the equation (1), i. e., the small angle scattering and Bethe approximation, and next the applicability of these approximations to the behaviour of proton and μ meson in liquid hydrogen.

The recoil momentum is defined by $q=p-p'$ which is gained by an atomic electron in a collision, where p and p' denote momenta of an incident particle before and after collision. The recoil momentum q distributes around the momentum q_0 which is gained by a perfect elastic collision between the incident particle and an electron at rest, to the extent of the mean square momentum of the atomic electron. For fast heavy particles the difference between q and q_0 is negligible compared with p, because it corresponds to the momentum of the atomic electron. In estimating q the case of the perfect head-on collision is relativistically considered:

$$\frac{q}{p}=\frac{2m(m+M)+2mE}{(m+M)^2+2mE}, \tag{3}$$

where M is the mass of the incident particle and E is its kinetic energy. The relation of reduced momentum transfer q/p versus E is shown in Fig. 1

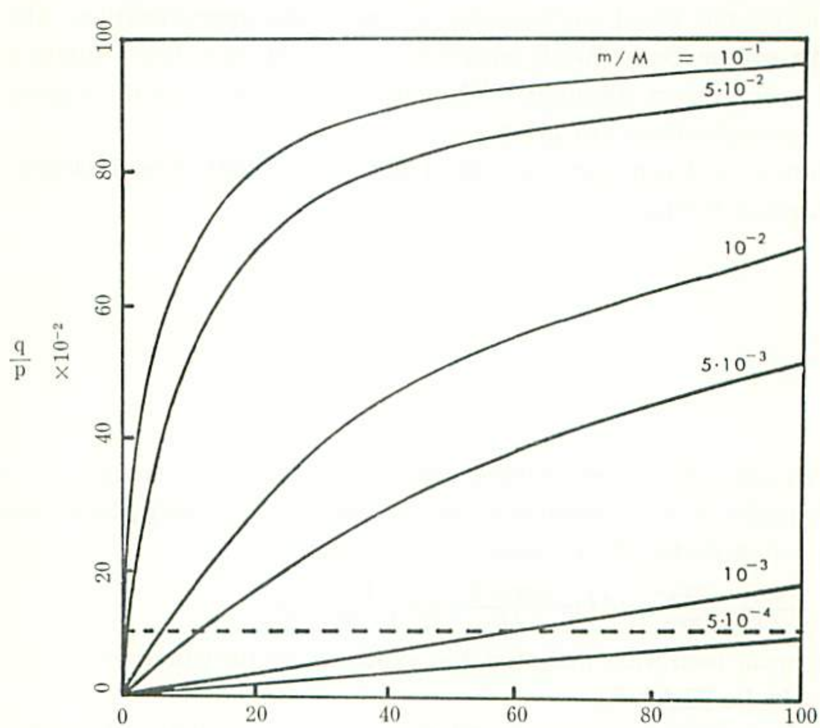

Fig. 1. Relation of redused momentum transfer q/p versus kinetic energy E of the incident particle, where q: momentum transfer, p: momentum of the incident particle, E: kinetic energy of the incident particle and M: mass of the incident particle.
The dashed line denotes $q/p=0.1$ as the criterion.

for the various values of m/M. The range of recoil momentum allowed on the small angle scattering approximation is assumed to $q/q<1/10$ and shown

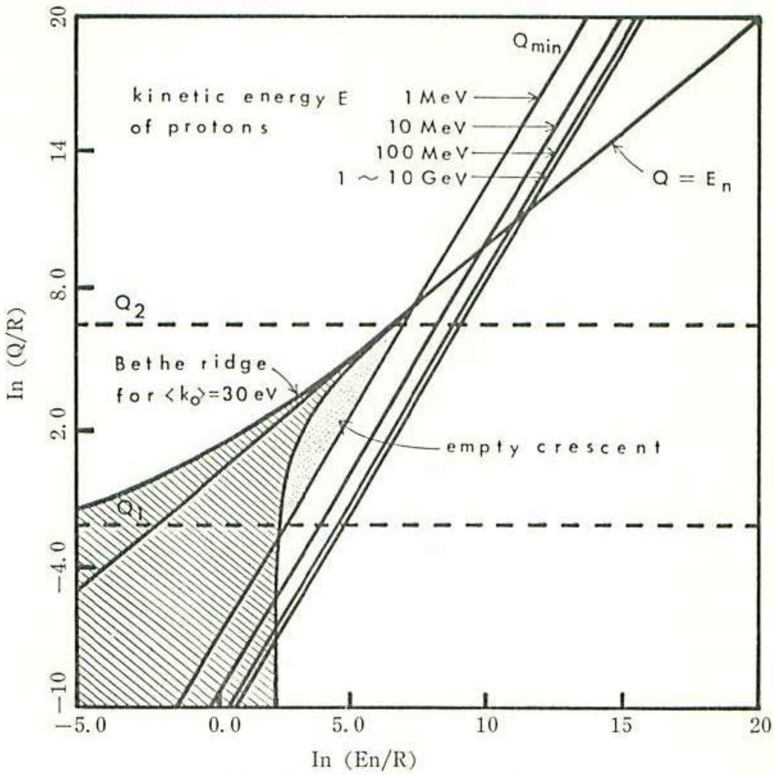

Fig. 2. Bethe ridge and Q_{min}.
The cross section of collision between incident particlesand a bound electron is mainly contributed from Bethe ridge.

with a horizontal dashed line in this figure. On this apporoximation allowed energy region is $E < 100\,\mathrm{GeV}$ for proton and $\mathrm{E} < 1\,\mathrm{GeV}$ for μ meson.

The differential cross section for the inelastic collision of these particles $d\sigma\ (E_n, Q)$ is a function of the excitation energy E_n of an atom and a parameter Q denoting the recoil energy of an electron defined by the equation

$$Q(1+Q/2mc^2)=q^2/2m.$$

This differential cross section is contributed mainly by the region Q smaller than $<k_o>$ for E_n smaller than $<k_o>$ and

$$\left|\frac{E_n-Q}{E_n}\right|^2 < \frac{4<k_o>}{E_n} \tag{4}$$

for large E_n, where $<k_o>$ stands for the average kinetic energy of atomic electrons[1]. The part of (E_n, Q)-plane which satisfies the latter condition is called Bethe ridge[2]. The condition (4) for $<k_o> =30\,\mathrm{eV}$ as twice binding energy of hydrogen atom and the lower limit of recoil energy derived from momentum conservation

$$Q_{\min}=E_n/2mc^2\beta^2-\text{relativistic terms} \tag{5}$$

are shown in Fig. 2. The stopping power is given by the integration of $E_n d\sigma\ (E_n, Q)$ over the allowed range $Q_{\min} < Q < Q_{\max}$ in the (E_n, Q)-plane, i. e.,

$$-\frac{dE}{ds}=\sum_n \int E_n d\sigma\ (E_n, Q). \tag{6}$$

The Bethe approximation stands on the existence of the recoil energies Q_1 and Q_2 of the electron that satisfy the following condition that both Q_1 and Q_2 are contained in the empty crescent, which is enclosed by the curves (5) and the lower edge of the Bethe ridge, and satisfy Q_1 smaller than $< k_o >$ and Q_2 much larger than $< k_o >$. The latter inequality is required to neglect the energy of bound electrons in comparison with the recoil energy above Q_2.

It is shown from Fig. 2 that these conditions are satisfied for proton energy more than one MeV, which corresponds to more than 0.045 in β. If β is used as an energy parameter, conversion to other particles may be easily done. For example, the same value of β corresponds to more than 100 keV for μ meson. This approximation becomes more better if the energy of the incident particle is higher, because smaller Q_1 and larger Q_2 are then allowed.

It is well known that Born approximation is a good one if the velocity of the bound electron is much smaller than that of the incident particle, because one can consider the interaction as a perturbation for the electron. The energy of bound electron is the order of several ten eV and the above condition is satisfied for the particles more than $\beta=0.045$. Therefore, the applicability of the Bethe-Bloch approximation in liquid hydrogen is restricted to

1 MeV $< E <$ 100 GeV for proton

and 100 keV $< E <$ 1 GeV for μ meson.

Literature

1) U. Fano, Ann. Rev. Nucl. Sci., **13**, 1 (1963).

2) M. Inokuchi, Rev. Mod. Phys., **43**, 297 (1971).